Final Report/Project: Solving PDEs

In many applications, using partial differential equations is the best way to model a scenario. They not only can show a change over time, but also the change as it relates to 2 or more independent variables. An couple examples of what they can model are heat and elasticity. The goal for this assignment is to find the area under a 3 dimensional curve given boundary values and equations. These are given as:

0 < x,y < 1

u(x, 0) = 1- 4(x-.5)2

u(x, 1) = 0

u(0, y) = 0

u(1, y) = 0

The way we are given to model this problem is using Laplace’s Equation, a modified form of Poisson’s Equation. Laplace’s Equation assumes the answer to be equal to zero. This gives us the equation

uxx + uyy = 0

Next, we apply finite difference methods. These are numerical methods that approximate solutions using derivatives. The methods can be applied to the second derivatives in such a way that they utilize the central difference method. You calculate this to become:

un(xj,yk) = ¼ [un(xj+1, yk) + un(xj-1, yk) + un(xj, yk+1) + un(xj, yk-1)]

The next step is to apply a mesh grid. The grid serves to give you reference points for your later calculations. The more reference points you have, the more accurate your calculations will be. The number of points you pick also should determine the dimensions of the matrix you are solving. If you have an NxN grid, your matrix should be (N-1)x(N-1), and the space between your points becomes h= 1/N = xj – xj-1